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SIMULATION OF TRANSFER OF SOLID PARTICLES BY A FILTRATION FLOW

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A mathematical model of transfer of solid particles by a filtration liquid flow is suggested. The phenomena of lowering the dynamic porosity and permeability of the porous medium by filling the intrapore space with a disperse phase are described. An equation is obtained that governs the change in the pore size distribution function in time.

Introduction. The process of transport of solid particles by a liquid in a porous medium is the concern of many investigations. A considreable portion of earlier publications dealt with simulation of transport of solid impurities through sand filters used for purification of water. At the present time, this problem has received much attention in petroleum production. When petroleum is displaced, the water injected into the bed usually carries different solid impurities in the form of disperse particles. The latter can appear in the filtration flow as a result of incomplete purification of the water before injection, from drill solutions that contain clay particles and penetrate into the beds, and from the porous medium itself on the surface of whose pores there are different solid particles that can be torn away by the flow. An adequate description of the processes of colmatation (the filling of the interstitial space with disperse particles) and suffosion (the separation of particles from the surface of the porous skeleton) is an important problem. In [1], a mathematical model of transport of particles by a single-phase flow is described that takes account of the processes of colmatation and suffosion, and corresponding kinetic relations are suggested. The same kinetic relationships are used in [2]. In the latter work the phenomenon of clogging up of pore channels with particles is also taken into account, and therefore the pore space is arbitrarily divided into two media, one of which contains channels that can be clogged, and the other contains channels that stay unclogged.

A more detailed description of the processes of colmatation, suffosion, and clogging of capillaries involves the size distribution function of the pores and a model representation of the porous medium. In [3], applying the results of [4] to evaluating colmatation, the dependence of the rate of narrowing of a pore channel on the dimensions of the capillary, the mean flow velocity in the channel, and the mean volume of the particles is presented. To evaluate the number of clogged pores, a probability approach is used. The change in the permeability is determined by means of an ideal model of the porous medium in the form of a bundle of capillaries.

Below we suggest a mathematical model of transport of disperse particles by a filtration flow. The porous medium is represented in the form of two interpenetrating continua [5, 6], one of which is connected with movable liquids and particles and the other with immovable ones. An equation is obtained that determines the dynamics of the pore size distribution function. The rate of change of the radius of the pore channel and the rate of decrease of the number of capillaries of a certain radius, which enter this equation, are evaluated proceeding from a model representation of the porous medium in the form of a bundle of capillaries with contractions. Corresponding expressions are obtained for the dynamic porosity, permeability, and mass exchange between the two media.

Mathematical Model. Suppose each point of the porous medium is characterized by the following quantities: porosity $m = m(x, y, z, t)$, absolute permeability $k^0 = k^0(x, y, z)$, and volume concentration of solid particles $C = C(x, y, z, t)$. Following [6], we arbitrarily divide the porous medium into two interpenetrating continua characterized by the porosities m_1 and m_2: $m_1 = m_1(x, y, z, t)$ is the portion of the pore space occupied by movable liquid; $m_2 = m_2(x, y, z, t)$ is the same with immovable liquid;

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$$m_1 + m_2 = m\,. \tag{1}$$

We represent the conservation equations for the first continuum in the form

$$\frac{\partial}{\partial t} m_1 + \operatorname{div} \mathbf{V} = -q\,, \tag{2}$$

$$\frac{\partial}{\partial t}(C_1 m_1) + \operatorname{div}(C_1 \mathbf{V} + D_u \operatorname{grad} C_1) = -q_c\,, \tag{3}$$

The equation of motion is written in the form of the Darcy law:

$$\mathbf{V} = -\frac{k_1}{\mu} \operatorname{grad}(P)\,, \tag{4}$$

The conservation equations for the second continuum are

$$\frac{\partial}{\partial t} m_2 = q\,, \tag{5}$$

$$\frac{\partial}{\partial t}(C_2 m_2) = q_c^{\eta}\,. \tag{6}$$

The concentration of particles in the first continuum is related to the concentration of particles in the second continuum by the obvious relation

$$Cm = C_1 m_1 + C_2 m_2\,. \tag{7}$$

To describe the mass exchange between the two continua and the changes in the filtration-capacity characteristics of the porous medium caused by deposition of particles on the walls of the capillaries and by clogging of a portion of the pore channels, we will use the pore size distribution function:

$$\eta = \varphi(r, t)\,. \tag{8}$$

For the initial instant of time $t = 0$ the pore-channel size distribution will be considered to be known, i.e.,

$$\varphi(r, 0) = \varphi^0(r)\,. \tag{9}$$

We rewrite Eq. (8) in the form

$$\eta - \varphi(r, t) = 0 \tag{10}$$

and take the total time derivative of Eq. (10). We obtain

$$-\frac{\partial \varphi}{\partial t} - \frac{\partial \varphi}{\partial r}\frac{dr}{dt} + \frac{d\eta}{dt} = 0 \tag{11}$$

or

$$\frac{\partial \varphi}{\partial t} + U_r \frac{\partial \varphi}{\partial r} + U_\eta = 0\,, \tag{12}$$

where $U_r = dr/dt$ is the rate of change of the radii of the pore channels; $U_\eta = -d\eta/dt$ is the rate of change of the number of pore channels of radius r. According to the physics of the phenomenon, the rate U_r is determined by the processes of colmation and suffosion, and the rate U_η, by the process of clogging of pore channels.

Thus, if the pore-channel size distribution $\eta = \varphi^0(r)$ for the initial instant of time is assigned and the rates $U_r(t)$ and $U_\eta(t)$ are known, then for any subsequent instant of time the pore-channel size distribution function will be determined by Eq. (12).

We note that colmatation is observed in all pore channels into which the suspension penetrated, whereas clogging of pores occurs only when the size of the particles is not smaller than the radii of the contractions (throats) r_s of the pore channels. Consequently, the rate U_η differs from zero only in the region $0 \le r_s \le R$, where $R = l/2$, and l is the characteristic size of the particles. We also note that channels the radii of whose throats were initially larger than R but, due to colmatation, whose dimensions came to satisfy the condition of clogging can also be clogged.

To evaluate the rate of contraction and clogging of the pore channels, we model an actual porous medium by a system of cylindrical capillaries of various radii that have pore contractions [7, 8]. We will assume that: 1) the particles are uniformly distributed in the liquid; 2) the ratio of throat radius to channel radius is the same for all capillaries and is maintained in the process of deposition of particles on the channel walls; 3) the volume of the throats is negligibly small compared to that of the cavities; 4) the additional hydraulic resistance caused by the presence of the throats is negligibly small; 5) a pore channel is completely blocked by a particle that entered the throat if the characteristic dimension of the particle is not smaller than the diameter of the throat. According to assumption 2, the throat size distribution can be characterized by the same distribution function as the pore size distribution. Assumption 5 can be weakened, namely, it can be assumed that the particle stuck decreases the diameter of the throat to a certain residual magnitude that is smaller than the characteristic dimension of a particle.

According to experimental data, the rate at which a cylindrical pore channel contracts due to colmatation can be calculated from the formula [3]

$$\frac{dr}{dt} = -C_1 \left(\frac{2u_s D^2}{rL}\right)^{1/3} . \tag{13}$$

The mean velocity in the pore channel u_s is related to the filtration velocity V by the relation

$$u_s = |\mathbf{V}| \, r^2/(8k_1) , \tag{14}$$

which can easily be obtained by combining the Poiseuille law for a capillary and the Darcy law for a porous-medium element represented by a bundle of capillaries.

In the time Δt, colmatation will change the radii of the capillaries by the value

$$\Delta r = U_r \Delta t , \tag{15}$$

which will lead to a reduction in the clearance. The new clearance (and, consequently, the porosity) will be of the form

$$m_1 (t + \Delta t) = m_1 \int_0^\infty \eta \, (r + \Delta r)^2 \, dr / \int_0^\infty \eta r^2 dr , \tag{16}$$

or, neglecting the term containing $(\Delta r)^2$ and taking into account Eq. (15), we obtain

$$m_1 (t + \Delta t) = m_1 \int_0^\infty \eta \, (r^2 + 2rU_r \Delta t) \, dr / \int_0^\infty \eta r^2 dr , \tag{17}$$

i.e., the clearance is changed by the value

$$\Delta m_1 = 2m_1 \int_0^\infty \eta r U_r \Delta t \, dr / \int_0^\infty \eta r^2 dr . \tag{18}$$

The total porosity of the bed m is changed by the same value. Having divided Eq. (18) by Δt and letting Δt tend to zero, we will have for m

$$\frac{\partial m}{\partial t} = 2m_1 \int_0^\infty rU_r \eta \, dr / \int_0^\infty r^2 \eta dr . \tag{19}$$

The intensity of the transition of the particles from the movable to the immovable state q_c^r due to colmatation will be

$$q_c^r = - \frac{\partial m}{\partial t} . \tag{20}$$

In order to evaluate the rate of clogging of the pore channels, we use assumption 2, according to which $r_s = hr$ (h is a certain constant identical for all channels). We consider channels having radii of the throats that satisfy the clogging condition:

$$r_s \leq R . \tag{21}$$

Let us assume that the portion of the capillaries that can be clogged, whose radii satisfy condition (21), is proportional to the number of particles that penetrated into such channels, with the proportionality factor β ($0 < \beta \leq 1$). In the time Δt a specimen of unit cross-sectional area admits

$$n = C_1 \, |\mathbf{V}| \, \Delta t / \Omega \tag{22}$$

particles, where Ω is the volume of a single particle. The number of particles that entered capillaries of radius r in this time will be proportional to the ratio of the cross-sectional area of the capillaries of radius r to the clearance area (this clearance area is equal to the dynamic porosity):

$$n_r = nN_r \pi r^2 / m_1 . \tag{23}$$

where N_r is the number of capillaries of radius r. The number of clogged capillaries will be equal to βn_r.

Let N be the total number of capillaries. Then the change in the pore size distribution function in the time Δt due to clogging can be calculated as

$$- \Delta \eta = \beta n_r / N = \frac{\beta n N_r \pi r^2}{N m_1} = \frac{\beta C_1 \, |\mathbf{V}| \, \Delta t \eta \, \pi r^2}{\Omega m_1} , \tag{24}$$

and the rate U_η will be equal to

$$U_\eta = \frac{- \Delta \eta}{\Delta t} = \frac{\beta C_1 \, |\mathbf{V}| \, \eta \, \pi r^2}{\Omega m_1} . \tag{25}$$

Thus, the coefficients U_r and U_η in Eq. (12) are defined by the dependences

$$U_r = - C_1 \left(\frac{|\mathbf{V}| \, r D^2}{4 k_1 L} \right)^{1/3} , \tag{26}$$

$$U_\eta = \begin{cases} \dfrac{\beta C_1 \, |\mathbf{V}| \, \eta \, \pi r^2}{\Omega m_1} & (2r \leq l/h) , \\ 0 & (2r > l/h) . \end{cases} \tag{27}$$

We evaluate the change in the absolute permeability caused by the influence of colmatation on the structure of the pore space by representing the permeability at the current instant of time $k_1(x, y, z, t)$ in the form of the product

$$k_1 = \bar{k}k^0, \tag{28}$$

where the coefficient $\bar{k}(x, y, z, t)$, which characterizes the relative change in the permeability of the first medium, is determined using the model of parallel capillaries and the Poiseuille law:

$$\bar{k} = \int_0^\infty r^4 \eta(r)\, dr \Big/ \int_0^\infty r^4 \varphi^0(r)\, dr. \tag{29}$$

The intensity of the transition of the liquid from the movable to the immovable state caused by clogging of the pore channels is determined by the volume of the clogged capillaries and can be calculated from the formula

$$q = \pi\beta C_1 \frac{|\mathbf{V}|}{\Omega} \int_0^{R/h} \eta r^4 dr \Big/ \int_0^\infty \eta r^2 dr \tag{30}$$

or

$$q = m_1 \int_0^{R/h} U_\eta r^2 dr \Big/ \int_0^\infty \eta r^2 dr. \tag{31}$$

The intensity of the transition of the particles to the immovable state because of clogging will be

$$q_c^\eta = C_1 q, \tag{32}$$

while the total intensity of the transition of the particles to the immovable state is

$$q_c = q_c^\eta + q_c^r. \tag{33}$$

NOTATION

m, porosity; m_1, dynamic porosity; m_2, portion of the pore space with immovable liquid; D_u, coefficient of convective diffusion; V, filtration velocity; P, pressure; k^0, absolute permeability; k_1, permeability of the first medium; μ, dynamic viscosity of the liquid; C, concentration of solid particles; C_1, C_2, concentration of particles in the first and second continua; r, radius of a pore channel; r_s, radius of the pore-channel throat; h, constant equal to the ratio of the throat radius to the pore-channel radius; t, time; η, portion of capillaries of radius r; φ, pore size distribution function; U_r, rate of change of the radii of the pore channels; U_η, rate of change of the number of pore channels of radius r; l, characteristic dimension of the particles; L, characteristic length of the pore channels; N, total number of capillaries in a sample with a unit cross-sectional area; N_r, number of capillaries of radius r; n, number of particles that penetrated into all the capillaries in a sample with a unit cross-sectional area; n_r, number of particles that penetrated into the capillaries of radius r; u_s, mean value of the liquid velocity in a channel; D, diffusion coefficient; q, intensity of the transition of the liquid from the movable to the immovable state; q_c^r, q_c^η, intensity of the transition of the particles from the movable to the immovable state because of deposition and clogging, respectively; q_c, total intensity of the transition of the particles to the immovable state. Subscripts: u, association of a quantity with the filtration velocity; c, association of a quantity with the concentration; s, mean value of a quantity.

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